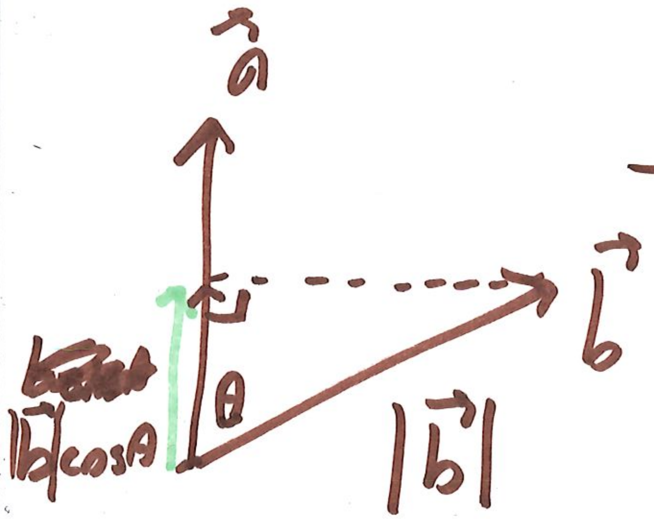
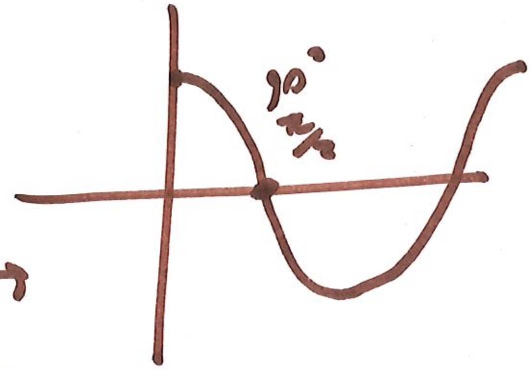
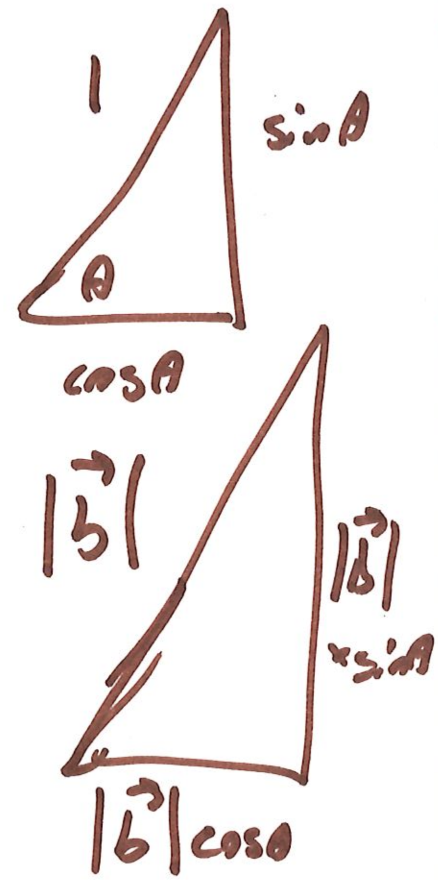
$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$ ← angle between $\vec{a}$ and $\vec{b}$







direction of $\vec{a}$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$|\vec{b}|\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

PROJECTION: OF $\vec{b}$ onto $\vec{a}$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a}$$

unit vector in the direction of $\vec{a}$ $\hat{a}$

a number

$\frac{1}{|\vec{a}|} \cdot \vec{a}$

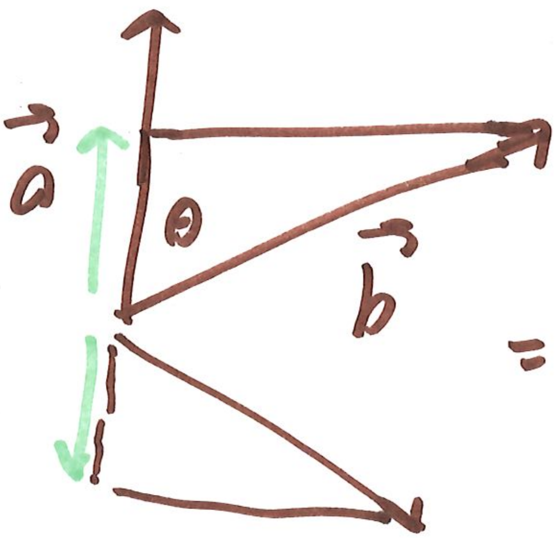
~~[illegible]~~: Ex. Find the projection of

$\vec{b} = (1, 0, 3)$ onto vector

$\vec{a} = (-2, 3, 1)$

$$\vec{a} \cdot \vec{b} = 1 \times (-2) + 0 \times (3) + 3 \times 1$$

$$= -2 + 0 + 3 = 1$$



length of projection

$$= \frac{1}{|\vec{a}|} = \frac{1}{\sqrt{14}}$$

new $\vec{a}'$ pointing same direction $(-4, 6, 2)$

projection should be same

$$\vec{a}' \cdot \vec{b} = (-4, 6, 2) \cdot (1, 0, 3) = -4 + 0 + 6 = 2$$

$$|\vec{a}| = \sqrt{(-2)^2 + 3^2 + 1^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$$

$$|\vec{a}'| = \sqrt{(-4)^2 + 6^2 + 2^2} = \sqrt{16 + 36 + 4} = \sqrt{56} = 2\sqrt{14}$$

length of projection is $\frac{2}{2\sqrt{14}} = \frac{1}{\sqrt{14}}$

$$\frac{1}{\sqrt{14}}\vec{a} = \left(-\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right)$$

$$\frac{1}{\sqrt{14}}\vec{a} = \left(-\frac{4}{\sqrt{14}}, \frac{6}{\sqrt{14}}, \frac{2}{\sqrt{14}}\right)$$

$$\frac{1}{\sqrt{14}}\frac{\vec{a}}{|\vec{a}|} = \left(-\frac{2}{14}, \frac{3}{14}, \frac{1}{14}\right)$$

$$\frac{1}{\sqrt{14}}\frac{\vec{a}}{|\vec{a}|} = \left(-\frac{4}{2\times 14}, \frac{6}{2\times 14}, \frac{2}{2\times 14}\right)$$

$\vec{a}=(-2,3,1)$ $\quad$ $\vec{b}=(1,0,3)$ $\quad$ $\frac{1}{14}\vec{a}$

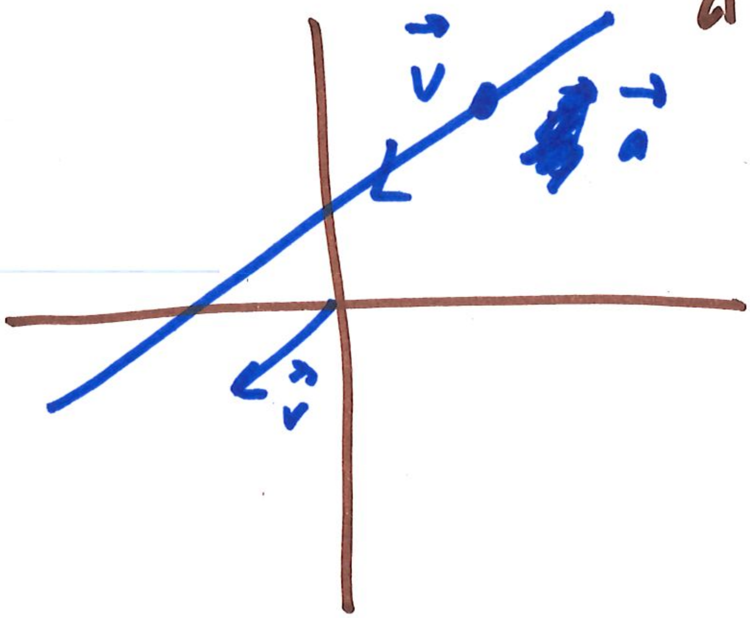
$|\vec{a}|=\sqrt{14}$ $\quad$ $|\vec{a}|^2=14$

$$=\frac{1}{14}(-2,3,1)$$

$$=\left(-\frac{2}{14}, \frac{3}{14}, \frac{1}{14}\right)$$

LINES $\vec{v}$ points in the direction of line, $\vec{a}$ on line

$$\text{line} = \{\vec{a} + t\vec{v} : t \text{ real number}\}$$



$\vec{a} = (a_1, a_2, a_3)$ $\vec{v} = (v_1, v_2, v_3)$

$$= \{(a_1 + tv_1, a_2 + tv_2, a_3 + tv_3) : t \in \mathbb{R}\}$$

"parametric"

parameter t

$$\cancel{t = \frac{-a_1}{v_1} \quad t = \frac{-a_2}{v_2} \quad t = \frac{-a_3}{v_3}}$$

$$x = a_1 + tv_1$$

$$t = \frac{x - a_1}{v_1} \quad t = \frac{y - a_2}{v_2} \quad t = \frac{z - a_3}{v_3}$$

$$\left\{(x, y, z) : \frac{x - a_1}{v_1} = \frac{y - a_2}{v_2} = \frac{z - a_3}{v_3}\right\}$$

PLANE

$$\{\vec{a} + s\vec{v} + t\vec{w};\ s, t \in \mathbb{R}\}$$

point (vector)

vectors giving 2 directions

normal vector $\vec{n} = \vec{v} \times \vec{w}$

SIMPLE CASE: plane contains $\vec{0}$

plane $= \{\vec{v} : \vec{v} \cdot \vec{n} = 0\}$

GENERAL CASE: plane contains $\vec{a}$

~~proj ont will be~~

proj onto $\vec{n}$ should always be the same