Determinant:

1×1 matrix $[3]$ $\det[3] = |[3]| = 3$

$[-2]$ $\det[-2] = |[-2]| = -2$

2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\det\begin{bmatrix} 1 & 3 \\ 7 & -2 \end{bmatrix} = 1 \cdot (-2) - 3 \cdot 7 = -2 - 21 = -23$$

3×3 matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = A$

$$\det A = a \det\begin{pmatrix} e & f \\ h & i \end{pmatrix} - b \det\begin{pmatrix} d & f \\ g & i \end{pmatrix} + c \det\begin{pmatrix} d & e \\ g & h \end{pmatrix}$$

$$= a(ei - fh) - b(di - fg) + c(dh - eg)$$

Ex

$$\det\begin{bmatrix}1&4&-3\\2&0&5\\-2&-3&1\end{bmatrix} = 1\cdot(0\cdot1-5\cdot(-3))$$

$$-4\cdot(2\cdot1-5\cdot(-2))$$

$$+(-3)(2\cdot(-3)-0\cdot(-2))$$

plus sign (not written)

minus sign

plus sign

$$=1\cdot15-4\cdot(2+10)$$

$$-3\cdot(-6)=15-48+18$$

$$=-15$$

CROSS PRODUCT $\vec{v}=(v_1, v_2, v_3)$

$\vec{w}=(w_1, w_2, w_3)$

$$\vec{i}=(1,0,0)$$

$$\vec{j}=(0,1,0)$$

$$\vec{k}=(0,0,1)$$

$$\det\begin{bmatrix}\vec{i}&\vec{j}&\vec{k}\\v_1&v_2&v_3\\w_1&w_2&w_3\end{bmatrix}$$

$$=\vec{i}(v_2w_3-v_3w_2)-\vec{j}(v_1w_3-v_3w_1)$$

$$+\vec{k}(v_1 w_2 - v_2 w_1)$$

$$= (v_2 w_3 - v_3 w_2, 0, 0) - (0, v_1 w_3 - v_3 w_1, 0)$$

$$+ (0, 0, v_1 w_2 - v_2 w_1)$$

$$= (v_2 w_3 - v_3 w_2, -v_1 w_3 + v_3 w_1, v_1 W_2 - v_2 w_1)$$

Ex $\vec{v} = (2, 3, 0)$ $\vec{w} = (-1, 2, 1)$

$$\det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 0 \\ -1 & 2 & 1 \end{bmatrix} = (3, -2, 4 - (-3))$$

$$= (3, -2, 7)$$

Triple Product $\vec{a}, \vec{b}, \vec{c}$

triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$

equiv. to $\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

Then $\vec{a} \times \vec{b}$ is perp. to both $\vec{a}$ and $\vec{b}$

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin \theta$$

θ angle between $\vec{a}$ and $\vec{b}$

$\vec{v}$ perp $\perp$ both $\vec{a}$ and $\vec{b}$

$$a_1 v_1 + a_2 v_2 + a_3 v_3 = 0 \qquad b_1 v_1 + b_2 v_2 + b_3 v_3 = 0$$

Torque