Dot product

$$(a_1, a_2)\cdot(b_1, b_2) = a_1b_1 + a_2b_2$$

$$(a_1, a_2, a_3)\cdot(b_1, b_2, b_3) = a_1b_1 + a_2b_2 + a_3b_3$$

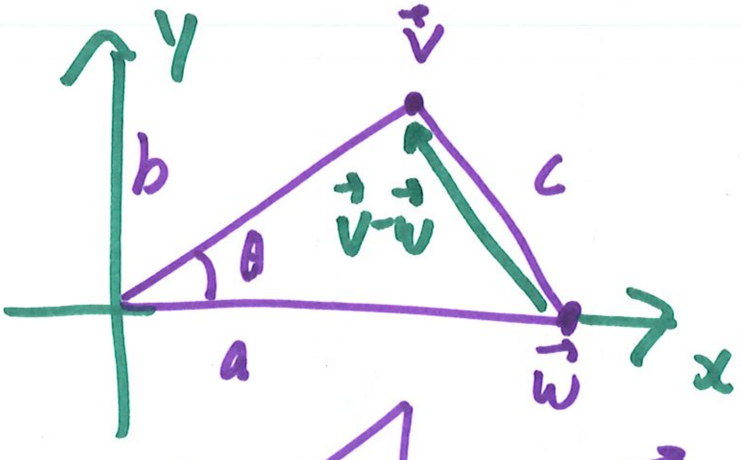
<u>Thm</u>: $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

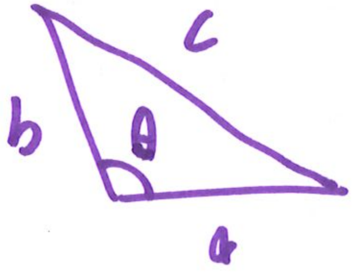
angle between $\vec{a}$ and $\vec{b}$

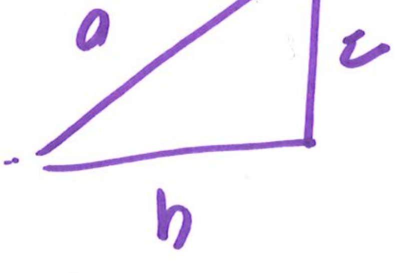
<u>Law of Cosines</u>: If a triangle has sides of lengths a, b, c, then

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

angle between sides of length a and b

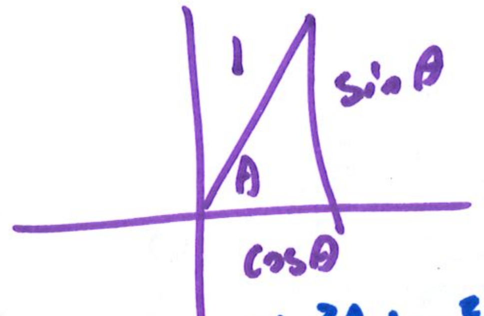






$$\vec{v} = (b\cos\theta, b\sin\theta)$$

dist. between $\vec{v}$ and $\vec{w}$



$\sin^2\theta + \cos^2\theta = 1$

$\vec{w} = (a, 0)$

$\vec{v} - \vec{w} = (b\cos\theta - a, b\sin\theta)$

$$(b\cos\theta - a)^2 + (b\sin\theta)^2 = b^2\cos^2\theta - 2ab\cos\theta + a^2$$

$$b^2\sin^2\theta$$

$$= b^2(\cos^2\theta + \sin^2\theta) - 2ab\cos\theta + a^2$$

$$= a^2 + b^2 - 2ab\cos\theta$$

$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$$

$$\shortparallel$$

$$(\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$$

length of $\vec{v}$ is $\sqrt{\vec{v} \cdot \vec{v}}$

$$= \vec{a} \cdot (\vec{a} - \vec{b}) - \vec{b} \cdot (\vec{a} - \vec{b})$$

$$= \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b}$$

$$= |\vec{a}|^2 + |\vec{b}|^2 - 2\,\vec{a} \cdot \vec{b}$$

$$-2\vec{a}\cdot\vec{b} = -2|\vec{a}||\vec{b}|\cos\theta$$

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

Ex. Find cosine between $(1,0,-3)$ and $(2,1.5,-1)$

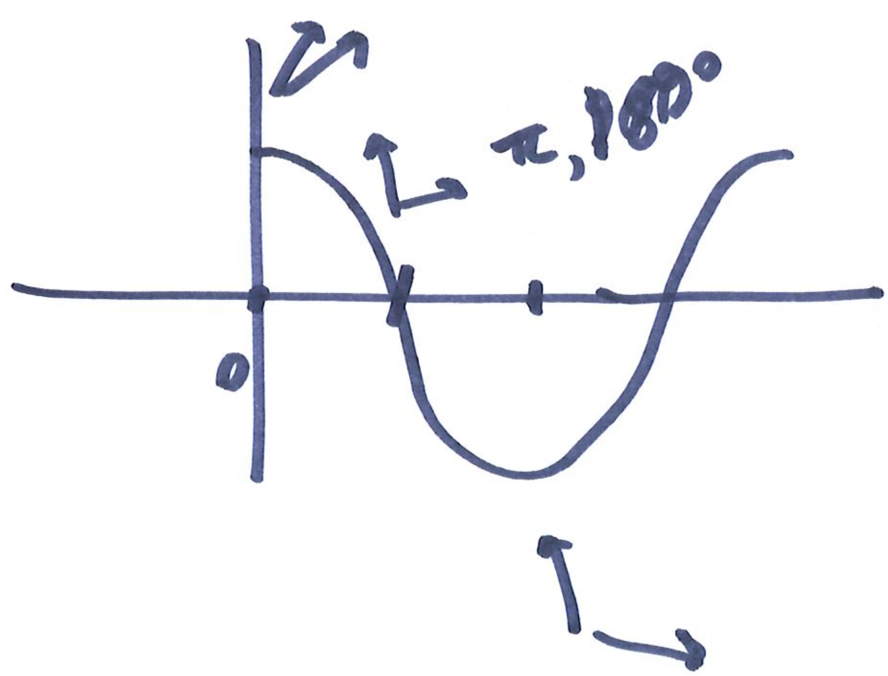
$$(1,0,-3)\cdot(2,1.5,-1) = 2+0+3 = 5$$

$$(1,0,-3)\cdot(1,0,-3) = 1+9 = 10 \quad \text{length} = \sqrt{10}$$

$$(2,1.5,-1)\cdot(2,1.5,-1) = 4+2.25+1 = 7.25$$

length $\sqrt{7.25}$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{5}{\sqrt{10}\sqrt{7.25}} \quad \blacksquare$$



Two vectors $\vec{u}, \vec{v}$ are perpendicular (orthogonal) if <u>and only if</u> $\vec{u} \cdot \vec{v} = 0$

Ex. $(1, 3, -2) \cdot (3, 1, 3) = 3 + 3 - 6 = 0$

Ex. $(1, -2.5, 3) \quad (-0.5, 2, z_2)$

want perp.

$-0.5 - 5 + 3z_2$ want $= 0$

$3z_2 = 5.5 \qquad z_2 = \frac{5.5}{3} \qquad \square$