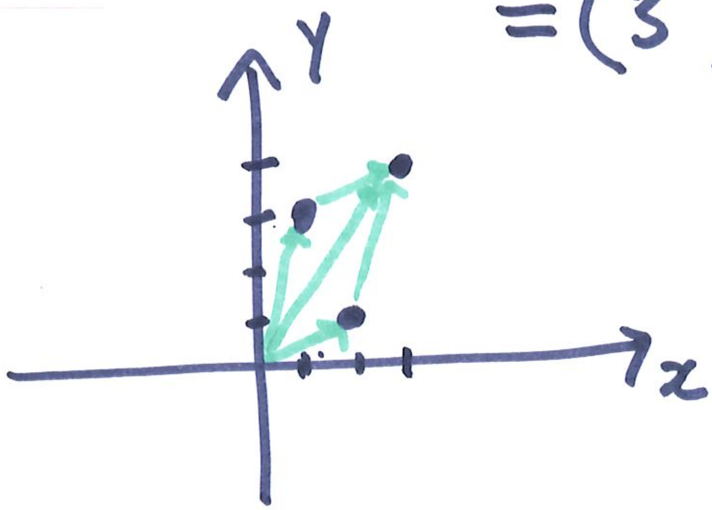
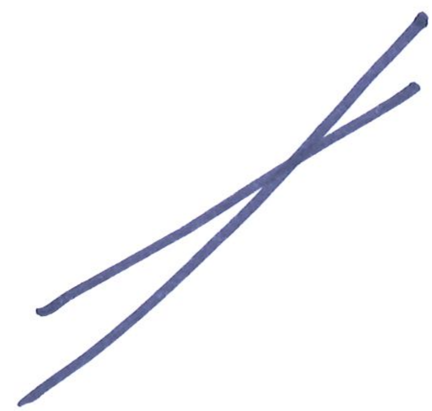
Vector Spaces

- add vectors

Example: $(2,1)+(1,3)$ $\begin{pmatrix}2\\1\end{pmatrix}+\begin{pmatrix}1\\3\end{pmatrix}$

$=(3,4)$



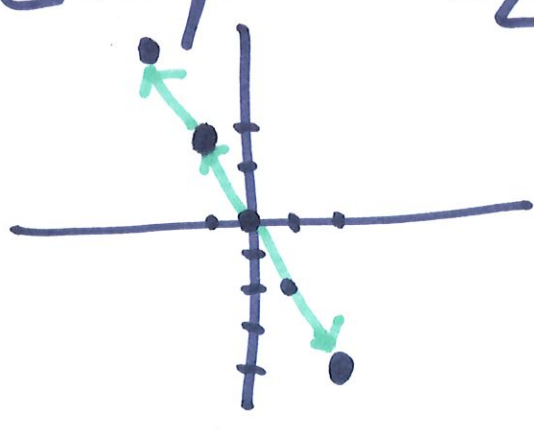


Example: $(-1, 0.5, 2.1)+(5.3, 7, -8.4)$

$=(4.3, 7.5, -6.3)$

- scalar multiplication

Example: $-2 \cdot (-1,2) = (2,-4)$



$2 \cdot (-1,2) = (-2,4)$

$-1 \cdot (-1,2) = (1,-2)$

Example: $\cancel{\frac{1}{2}(}$ $\frac{1}{2}(3, 1.4, -2) = (\frac{3}{2}, .7, -1)$

1. addit'n is commutative $a+b=b+a$
2. " " associative $(a+b)+c = a+(b+c)$
3. " has an identity element $\vec{a} + \underline{\vec{0}} = \vec{a}$

$(0,0)$ $(0,0,0)$

4. addit'n has an inverse $-\vec{a} + \vec{a} = \vec{0}$

$(-1,2)$ $(1,-2) = -(-1,2)$

5. scalar multiplication distributes over addition

vectors

scalar $\longrightarrow$ $c(\vec{v} + \vec{w}) = c\vec{v} + c\vec{w}$

6. "

vector

scalars $(a+b)\vec{v} = a\vec{v} + b\vec{v}$

7. scalar multiplication is associative

$b(a\vec{v}) = (ba)\vec{v}$

scalars vector

8. scalar mult. has an identity

$1\vec{v} = \vec{v}$

ordinary number 1

DOT PRODUCT

Example: $(1,0,-3)\cdot(2,1.5,-2)$

$1\cdot 2 + 0\cdot 1.5 + (-3)\cdot(-2) = 2+0+6 = 8$

Properties

- $\vec{v}\cdot\vec{v}$ is positive (or 0)

  $(1,-2,0)\cdot(1,-2,0) = 1+4+0 = 5$

  only 0 if $\vec{v} = \vec{0}$

- commutative $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

- distributes over addition

  $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$

scalar

- $(c\vec{u})\cdot\vec{v} = c(\vec{u}\cdot\vec{v}) = \vec{u}\cdot(c\vec{v})$

vectors

- dot product with $\vec{0}$ is 0

$$\vec{v} \cdot \vec{0} = 0$$

Theorem: $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$

angle between $\vec{a}$ and $\vec{b}$

Uses Law of Cosines